KINEMATIC ANALYSIS OF ROBOT

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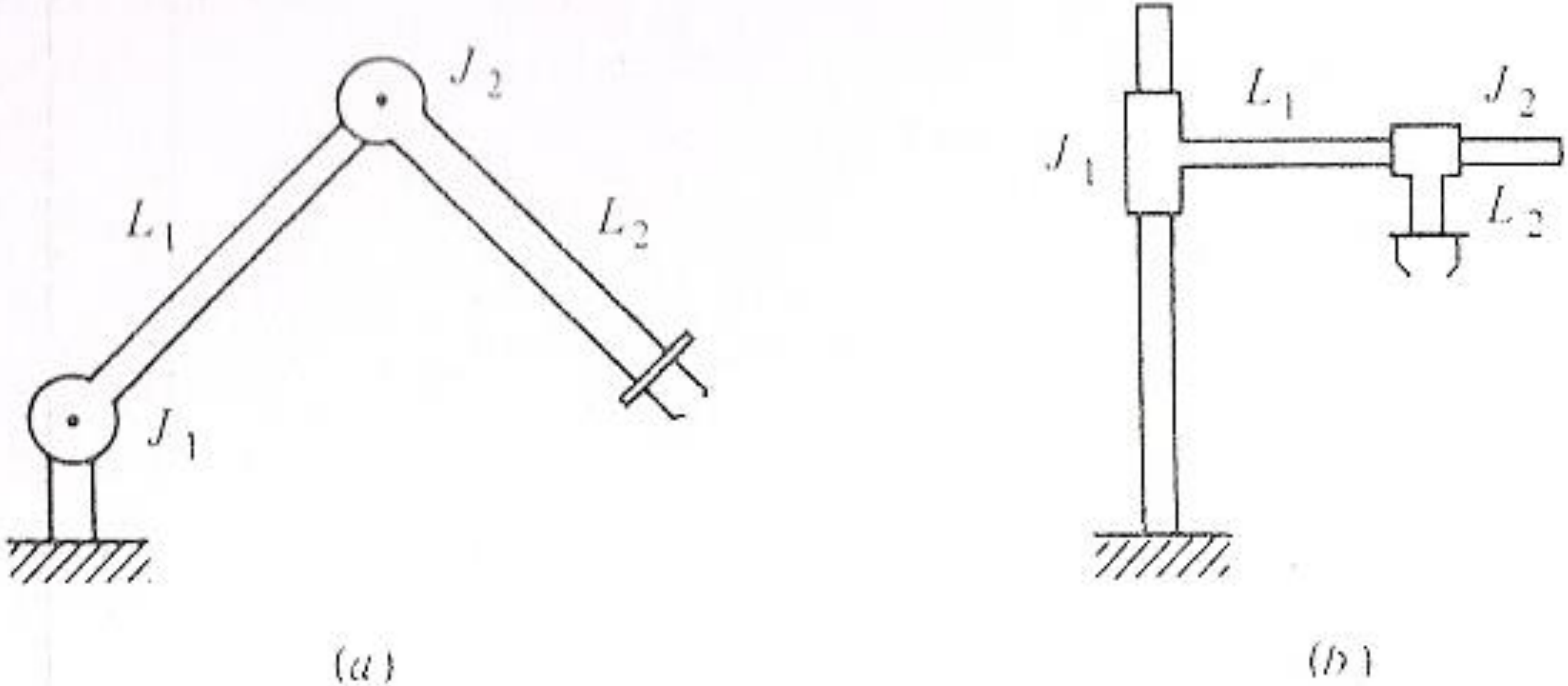

Figure 4-1 Two different 2-jointed manipulators. (a) two rotational joints (RR), (b) two linear joints (LL).

POSITION REPRESENTATION

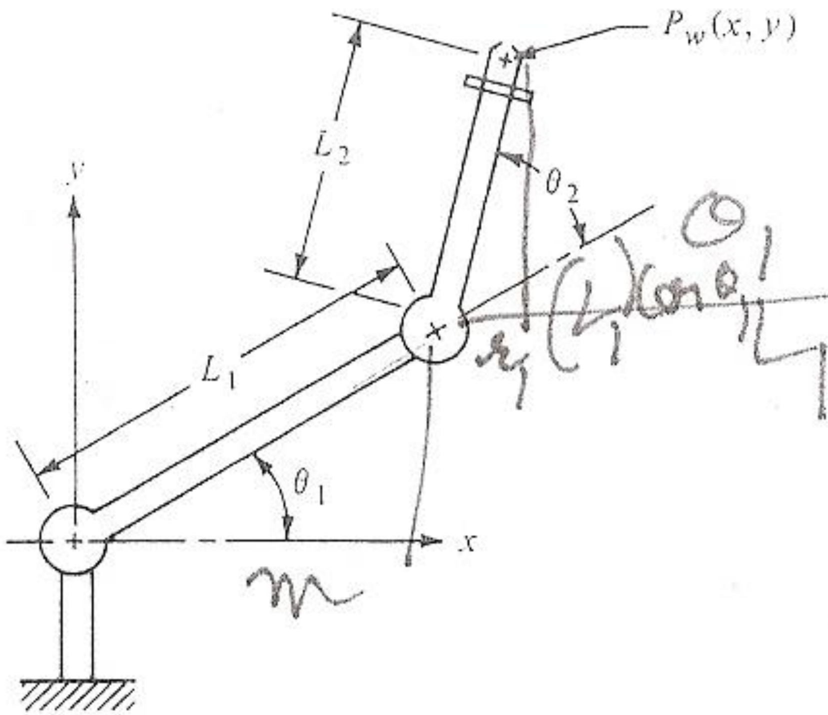

Figure 4-2 A two-dimensional 2 degree-of-freedom manipulator (type RR).

$$P_w = (x, y)$$

Forward Transformation of a 2-Degree of Freedom Arm

We can determine the position of the end of the arm in world space by defining a vector for link 1 and another for link 2.

$$\mathbf{r}_1 = [L_1 \cos \theta_1, L_1 \sin \theta_1] \tag{4-1}$$

$$\mathbf{r}_2 = [L_2 \cos(\theta_1 + \theta_2), L_2 \sin(\theta_1 + \theta_2)] \tag{4-2}$$

Vector addition of (4-1) and (4-2) yields the coordinates x and y of the end of the arm (point P_w) in world space

$$x = L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) \tag{4-3}$$

$$y = L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2) \tag{4-4}$$

Reverse Transformation of the 2-Degree of Freedom Arm

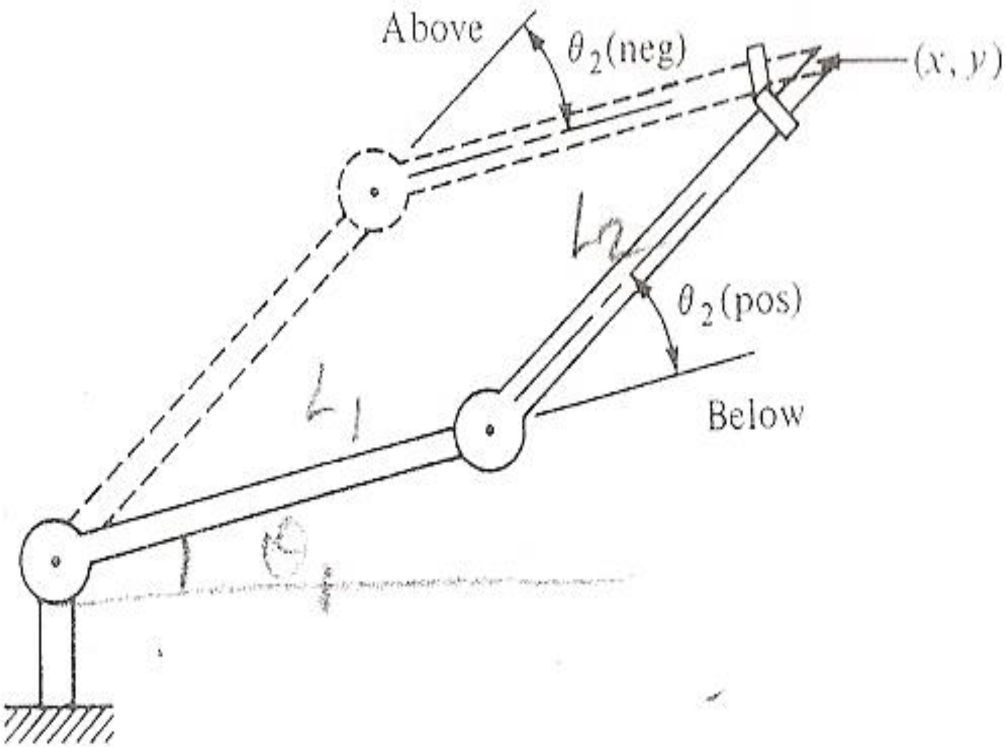

Figure 4-3 The arm at point $P(x, y)$, indicating two possible configurations to achieve the position.

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

we can rewrite Eqs. (4-3) and (4-4) as

$$x = L_1 \cos \theta_1 + L_2 \cos \theta_1 \cos \theta_2 - L_2 \sin \theta_1 \sin \theta_2$$

$$y = L_1 \sin \theta_1 + L_2 \sin \theta_1 \cos \theta_2 + L_2 \cos \theta_1 \sin \theta_2$$

Reverse Transformation of the 2-Degree of Freedom Arm

Squaring both sides and adding the two equations yields

$$\cos\theta_2 = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2} \tag{4-5}$$

Defining α and β as in Fig. 4-4 we get

$$\tan\alpha = \frac{L_2\sin\theta_2}{L_2\cos\theta_2 + L_1}$$

$$\tan\beta = \frac{y}{x} \tag{4-6}$$

Using the trigonometric identity

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

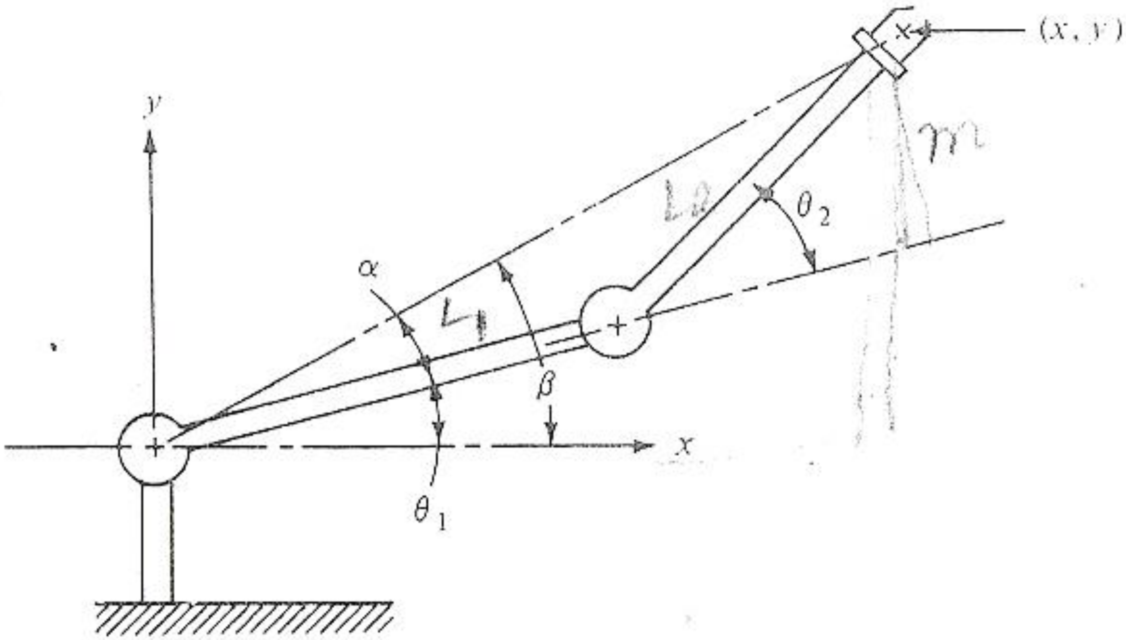

Figure 4-4 Solving for the joint angles.

we get

$$\tan\theta_1 = \frac{[y(L_1 + L_2\cos\theta_2) - xL_2\sin\theta_2]}{[x(L_1 + L_2\cos\theta_2) + yL_2\sin\theta_2]} \qquad (4\text{-}7)$$

Knowing the link lengths L_1 and L_2 we are now able to calculate the required joint angles to place the arm at a position (x, y) in world space.

Adding Orientation: A 3-Degree of Freedom Arm in Two Dimensions

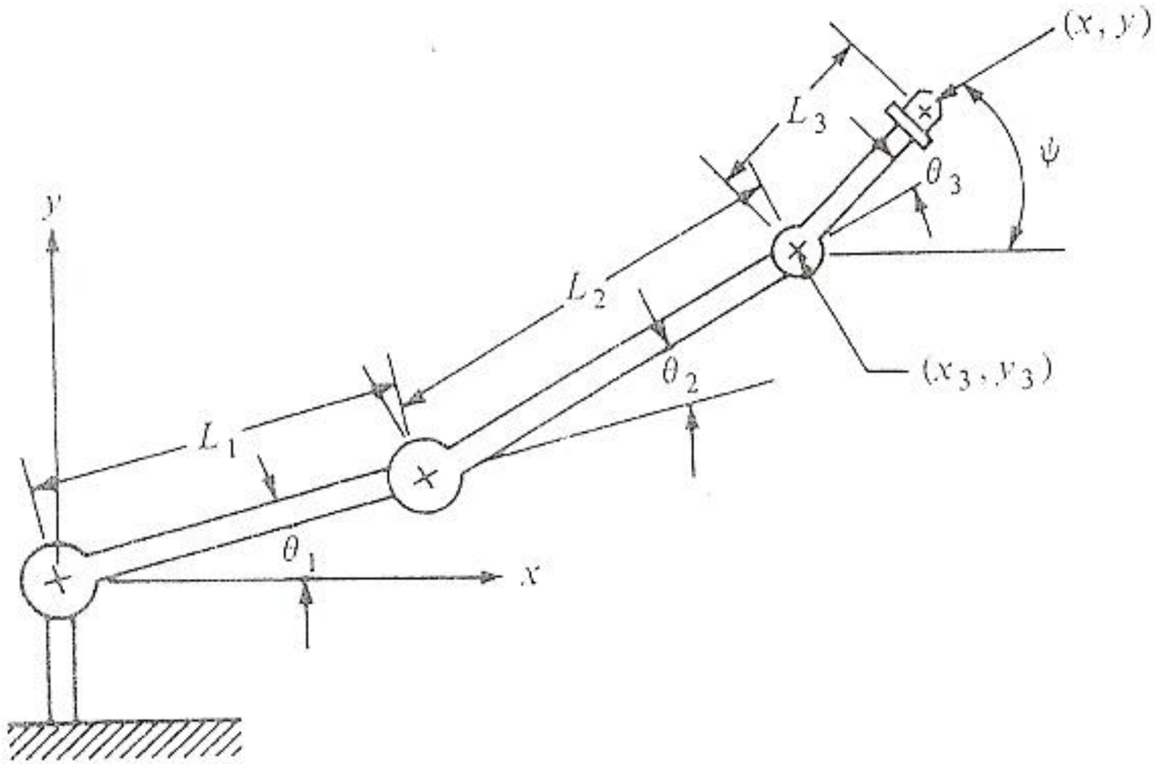

Figure 4-5 The two-dimensional 3 degree-of-freedom manipulator with orientation (type RR:R).

Adding Orientation: A 3-Degree of Freedom Arm in Two Dimensions

The arm we have been modeling is very simple; a two-jointed robot arm has little practical value except for very simple tasks. Let us add to the manipulator a modest capability for orienting as well as positioning a part or tool. Accordingly, we will incorporate a third degree of freedom into the previous configuration to develop the *RR*:*R* manipulator shown in Fig. 4-5. This third degree of freedom will represent a wrist joint. The world space coordinates for the wrist end would be

$$\left.\begin{aligned} x &= L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ y &= L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3) \\ \psi &= (\theta_1 + \theta_2 + \theta_3) \end{aligned}\right\} \quad (4\text{-}8)$$

We can use the results that we have already obtained for the 2-degree of freedom manipulator to do the reverse transformation for the 3-degree of freedom arm. When defining the position of the end of the arm we will use x, y, and ψ. The angle ψ is the orientation angle for the wrist. Given these three values, we can solve for the joint angles (θ_1, θ_2, and θ_3) using

$$x_3 = x - L_3 \cos \psi$$

$$y_3 = y - L_3 \sin \psi$$

Having determined the position of joint 3, the problem of determining θ_1 and θ_2 reduces to the case of the 2-degree of freedom manipulator previously analyzed.

A 4-Degree of Freedom Manipulator in Three Dimensions

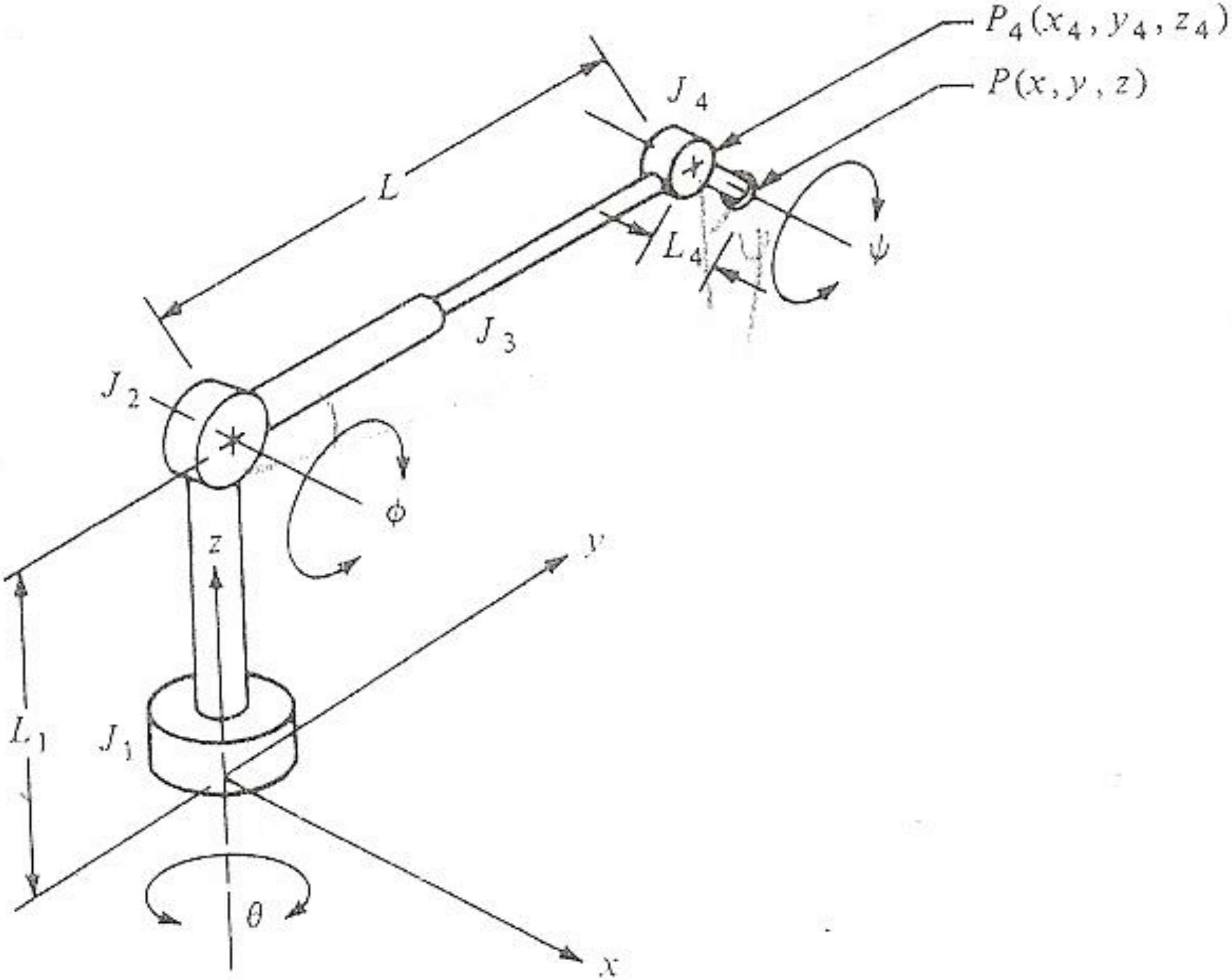

Figure 4-6 A three-dimensional 4 degree-of-freedom manipulator (type TRL:R).

A 4-Degree of Freedom Manipulator in Three Dimensions

Figure 4-6 illustrates the configuration of a manipulator in three dimensions. The manipulator has 4 degrees of freedom: joint 1 (type T joint) allows rotation about the z axis; joint 2 (type R) allows rotation about an axis that is perpendicular to the z axis; joint 3 is a linear joint which is capable of sliding over a certain range; and joint 4 is a type R joint which allows rotation about an axis that is parallel to the joint 2 axis. Thus, we have a $TRL{:}R$ manipulator.

Let us define the angle of rotation of joint 1 to be the base rotation θ; the angle of rotation of joint 2 will be called the elevation angle ϕ; the length of linear joint 3 will be called the extension L (L represents a combination of links 2 and 3); and the angle that joint 4 makes with the $x-y$ plane will be called the pitch angle ψ. These features are shown in Fig. 4-6.

The position of the end of the wrist, P, defined in the world coordinate system for the robot, is given by

$$x = \cos\theta(L\cos\phi + L_4\cos\psi) \tag{4-9}$$

$$y = \sin\theta(L\cos\phi + L_4\cos\psi) \tag{4-10}$$

$$z = L_1 + L\sin\phi + L_4\sin\psi \tag{4-11}$$

Given the specification of point P (x, y, z) and pitch angle ψ, we can find any of the joint positions relative to the world coordinate system. Using P_4 (x_4, y_4, z_4), which is the position of joint 4, as an example,

$$x_4 = x - \cos\theta(L_4 \cos\psi) \tag{4-12}$$

$$y_4 = y - \sin\theta(L_4 \cos\psi) \tag{4-13}$$

$$z_4 = z - L_4 \sin\psi \tag{4-14}$$

The values of L, ϕ, and θ can next be computed:

$$L = [x_4^2 + y_4^2 + (z_4 - L_1)^2]^{-1} \tag{4-15}$$

$$\sin\phi = \frac{z_4 - L_1}{L} \tag{4-16}$$

$$\cos\theta = \frac{y_4}{L} \tag{4-17}$$

The example we have just done is simple but not unrealistic. In order for a robot controller to be able to perform the calculations necessary quickly enough to maintain good performance they must be kept as simple as possible. The manipulator kinematics described in this example are very similar to those of the MAKER robot, by U.S. Robots. The only real difference is that the MAKER's wrist mechanism has more than a single joint.

One facet of our approach in the preceding analysis which should be noted by the reader is that we separated the orientation problem from the positioning problem. This approach of separating the two problems greatly simplifies the task of arriving at a solution.

THANK YOU FOR YOUR PATIENCE AND TIME